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Grounds for the renewal and mathematisation of logic and their historical characterisation

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Abstract. The relevance of the study is conditioned by the necessity of a historico-philosophical understanding of the process of renewal and mathematisation of logic, which determined the formation of contemporary mathematical logic and substantially influenced the development of the philosophy of science, mathematics, and the analytic tradition. The aim of the article was to elucidate the philosophical grounds, arguments, and conceptual prerequisites that contributed to the reform of classical logic and the emergence of its mathematical forms in the second half of the nineteenth and the early twentieth centuries. The methodological basis of the study comprised the historical, analytical, comparative, and genealogical methods, which allow one to trace the evolution of conceptions of logic from the scholastic tradition to the modern forms of mathematical logic. The method of formalisation was also employed to analyse the distinctive features of propositional logic and predicate logics in the context of their differences. As a result of the study, it was established that the first grounds for the modernisation of logic were formed already within the scholastic tradition, in particular in the works of W. Ockham, where the necessity of refining and simplifying syllogistic rules was substantiated. It was shown that an important stage in the development of these ideas was constituted by the logical projects of G.W. Leibniz, who was among the first to advance a programme for the mathematisation of logic and the creation of a universal symbolic language of science. It was determined that a decisive role in the reform of logic was played by the investigations of G. Frege, who laid the foundations of logicism, carried out the axiomatisation of propositional logic, and created the foundations of the modern predicate calculus. It was established that the further development of these ideas in the works of B. Russell and A.N. Whitehead contributed to the consolidation of mathematical logic as an independent branch of knowledge and a universal instrument of scientific analysis. It was also established that important factors in the mathematisation of logic were the limitations of classical syllogistic and the necessity of overcoming the logical paradoxes that arose in the course of the

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development of mathematics. The practical value of the study lies in the possibility of using its results for the historico-philosophical analysis of logic, for teaching courses in logic, philosophy of science, and the history of analytic philosophy, as well as for further research into the genesis of mathematical logic

Keywords: positivism; logicism; cognition; syllogistic; predicate calculus; quantifier; judgement

Introduction

The study of the history and genealogy of logic remains a relevant direction of contemporary historico-philosophical scholarship, since it allows one to trace the formation of those conceptual foundations that have determined the modern understanding of logic, mathematics, and scientific method. This problematic acquires particular significance in connection with the active development of formal systems, artificial intelligence, and computer science, the foundation of which is precisely mathematical logic. At the same time, an understanding of the process of its formation requires recourse not only to the history of mathematics but also to the philosophical prerequisites for the reconstruction of classical logic. Amongst contemporary studies, considerable attention is drawn to the work of J. Lemanski (2025), which analyses the very possibility of a historical approach to logic. The author criticised the positions of logical monism, according to which there exists only one universal logic, and shows that the development of logical knowledge must necessarily be regarded as a historical process of transformation of logical systems and conceptions. In the work of Z. Rybaříková (2024), the significance of contemporary formal methods for the history of logic is examined. The author demonstrated that the application of mathematical logic to the analysis of historical logical theories allows for a reinterpretation of the development of logical thought and a reconsideration of the place of classical logic in the historical process.

A separate group of studies consists of works devoted to the legacy of G. Frege. E.D. Walker & E.H. Reck (2024) analysed the specifics of G. Frege's notation and its role in the formation of contemporary formal logic. The authors showed that the innovation of G. Frege lay not only in the creation of a new logical apparatus but also in a fundamentally new mode of representing logical relations. Likewise, in an editorial article, E.H. Reck (2024) examines the place of G. Frege in the history of logic and emphasises that it is the Begriffsschrift that is traditionally regarded as the starting point of modern logic. At the same time, the author underscored the necessity of a broader historical context for understanding the significance of Frege's reform of logic. In the study of G. Eder (2023), the distinctive features of Frege's understanding of logical consequence and its significance for the contemporary philosophy of logic are analysed. The work demonstrated that the legacy of G. Frege continues to remain the subject of active theoretical debate. In the article of K.B. Willert (2024), the normative status of logic in G. Frege's philosophy is examined. The author showed that logic in G. Frege is conceived not merely as a formal system but also as the foundation of rational thought and scientific cognition. In the work of G. Baratelli (2024), the philosophical consequences of the transformation of logic in the twentieth century are examined. G. Baratelli showed that the formation of modern logic substantially altered the very understanding of science and scientific rationality. This attests to the relevance of investigating the historical origins of those logical systems that today constitute the foundation of a considerable portion of scientific knowledge.

Notwithstanding the considerable number of contemporary studies, the majority of them focus either on particular aspects of G. Frege's work or on specialised questions of contemporary logic and the philosophy of logic. At the same time, the problem of a holistic reconstruction of the philosophical grounds for the renewal and mathematisation of logic as a prolonged historical process – one that takes its origin in the scholastic tradition, passes through the logical projects of G.W. Leibniz, and culminates in the formation of contemporary mathematical logic in the works of G. Frege, A.N. Whitehead, and B. Russell – remains insufficiently elucidated. It is precisely this research lacuna that determined the aim of the present article, which consisted in the analysis of the philosophical arguments and positions that were embodied as grounds and foundations in the process of the renewal and mathematisation of logic.

Materials and Methods

The object of the study was the historical development of logic as a philosophical and scientific discipline in the context of its modernisation and mathematisation. The subject of the study was the philosophical grounds, conceptual prerequisites, and methodological principles that contributed to the transformation of classical logic into contemporary mathematical logic. The source base of the study comprised primary and secondary sources. The primary sources included the logical and philosophical works of C.S. Peirce (1873), G. Frege (1903), W. Ockham (1957), and other thinkers whose investigations played a defining role in the formation of contemporary logic. The secondary sources are represented by contemporary historico-philosophical studies devoted to the history of logic, philosophy of mathematics, logicism, and the genesis of formal systems, in particular P. Mancosu (2008), I.V. Khomenko (2010), and W. Ewald (2019). The selection of sources was determined by the necessity of reconstructing the genealogy of mathematical logic and identifying the principal philosophical grounds that determined its emergence and subsequent development.

The methodological basis of the study comprised a complex of interrelated methods of historico-philosophical and logico-conceptual analysis, applied for the reconstruction of the philosophical grounds for the renewal and mathematisation of logic and for the elucidation of the genealogy of this process from the scholastic tradition to the formation of contemporary mathematical logic. The chronological scope of the study encompasses the period from the late Middle Ages and the early modern period to the end of the nineteenth and the beginning of the twentieth century, when the decisive transformation of classical logic into mathematical logic took place. The historico-philosophical method was employed for the reconstruction of the intellectual context in which the projects of reforming logic were formed and developed. The application of this method allowed one to trace the evolution of conceptions concerning the nature, functions, and purpose of logic, as well as to identify the philosophical prerequisites that determined the attempts to revise the traditional Aristotelian syllogistic. Historico-philosophical analysis also made it possible to establish the conceptual continuity between the logical investigations of scholasticism, G.W. Leibniz's project of a universal symbolic language, and the logical innovations of the second half of the nineteenth century.

The comparative method was applied for the analysis of various approaches to the modernisation of logic and for identifying the commonalities and differences between classical syllogistic and contemporary formal logical systems. Particular attention was paid to a comparison of the logical projects of W. Ockham, G.W. Leibniz, G. Frege, A.N. Whitehead, and B. Russell. Comparative analysis allowed one to identify both elements of continuity and fundamentally new ideas that contributed to the mathematisation of logic and the formation of predicate calculi. The method of conceptual analysis was employed to clarify the content and functions of the key concepts of the study, in particular such concepts as "mathematisation of logic", "logicism", "propositional logic", "predicate logic", "formal system", and "logical calculus". The application of this method made it possible to identify the conceptual transformations that accompanied the transition from traditional logic to the modern forms of mathematical logic. The method of logical reconstruction was employed for the systematisation of philosophical arguments regarding the necessity of reforming logic and for the explication of the theoretical foundations of the development of mathematical logic. This method allowed one to reproduce the internal logic of the argumentation of the principal representatives of the modernisation of logic and to demonstrate the interrelation between mathematical innovations and philosophical conceptions of the nature of scientific knowledge.

Results and Discussion

The understanding of the path of development of logic in scholarly discourse and the first grounds for its modernisation

The history of logic as a science of the laws, forms, and principles of correct reasoning spans more than two and a half millennia. Throughout this time, the syllogistic logic formed by Aristotle remained the dominant logical system, although individual propositions thereof were repeatedly subjected to refinement and reconceptualisation. At the same time, contemporary mathematical logic differs substantially from the classical tradition both in its subject matter and in its methods of inquiry. Despite the centuries-long history of the development of logic, its transformation into a formalised mathematical discipline occurred only in the mid-nineteenth and early twentieth centuries. A decisive role in this process was played by the works of G. Boole, A. de Morgan, C.S. Peirce, E. Schröder, G. Frege, G. Peano, B. Russell, D. Hilbert, G. Cantor, and K. Gödel. As J. van Heijenoort (1967) and W.C. Kneale & M. Kneale (1962) noted, it is expedient to examine the history of logic on the basis of two stages of its development. The first stage spans from the very emergence of Aristotelian logic to the mid-nineteenth century, and the second from the mid-nineteenth century to the present day. A similar understanding of the development of logic as a science in a historico-philosophical context is presented in the foundational works of H. Scholz (1931), P.J. Hurley & L. Watson (2018), as well as in the study of the neo-positivist V. Kraft (1953). Amongst contemporary Ukrainian scholars, similar positions are also expressed in the works of I.V. Khomenko (2010) and Y. Shramko (2005b). The first stage of the development of logic was characterised by the relative stability of its subject field: the principal propositions, methods, and range of problems remained, to a considerable degree, connected with the Aristotelian logical tradition. I. Kant, for instance, regarded formal logic as a fairly complete science that had made no progress since antiquity. After the reform of logic, such positions were refuted, but even prior to this, attempts at its improvement can be identified in the history of logic – attempts whose foundations rested on the aspiration to clarify and refine classical syllogistic, as was indicated by V. Goranko (2023).

Turning to the era of scholasticism, one should recall W. Ockham and his follower A. of Saxony, who engaged with the internal semantics of formal logic and, being clearly aware of all its complexity and unwieldiness, sought to improve Aristotle's syllogistic by simplifying and refining its rules and principles; and it is precisely in the work of W. Ockham and A. of Saxony that one first encounters systematic attempts to modernise formal logic on the basis of ideas concerning its refinement and clarification (Goranko, 2023). It was precisely these ideas of refining and simplifying classical syllogistic that became the foundation of W. Ockham's work in the domain of logic. He develops a consistent critique of the excessive complexity and elaborateness of syllogistic rules, which, in his view, needed to be simplified and made more comprehensible. He maintained that syllogisms must be grounded in clear and definite terms and that ambiguity in the definition of terms must be avoided. In the Summa logicae, W. Ockham (1957) also proposed his own rules for working with syllogisms – for instance, he held that a simple categorical syllogism must not contain more than two terms and that two negative terms cannot be used within a single syllogism.

W. Ockham (1957) regarded logic as one of the most important instruments of cognition, without which no science can be fully comprehended. In his view, unlike material instruments, logic does not lose its value in the course of use; on the contrary, it develops and is perfected through its application across different fields of knowledge. W. Ockham made a substantial contribution to the logic of his era, and it is in his works and approaches that the ideas of clarifying and refining classical logic gained wide currency during the period of scholasticism. One may discern in this a precondition for the subsequent application of the fundamental principles of mathematics to logic in the context of its refinement and expansion of its possibilities. It is for this reason that certain intentions and elements of scholastic logic may be regarded as one of the historical prerequisites for the further modernisation and mathematisation of logic.

At the same time, the early attempts at the reform of logic did not lead to its mathematisation in the modern sense. The principal reason for this was the absence of the necessary mathematical apparatus and of sufficiently developed formal means for describing logical relations. Scholastic logic focused primarily on the analysis of concepts and syllogisms, whilst the instruments for the symbolic representation of complex logical structures had yet to be created. Besides W. Ockham and A. of Saxony, R. Llull, J. Buridan, P. of Spain, W. Heytesbury, and representatives of the late Ockhamist tradition (including G. of Rimini and the Parisian school of the fourteenth century) may also be counted amongst the forerunners of the tendencies towards formalisation and quasi-mathematisation of logic. Similar historiographical positions may be found in W.C. Kneale & M. Kneale (1962), in particular in the sections devoted to Ockham, Buridan, and the Parisian school of the fourteenth century. The works of other lesser-known representatives of the scholastic tradition, alongside the principal nominalists in the manner of Ockham, likewise demonstrated a persistent aspiration towards the algorithmisation of inference, the refinement of logical relations, and the formal systematisation of argumentation, which created an important proto-formal stratum for the subsequent development of symbolic logic in the nineteenth century. It was precisely these scholastics that played an important preparatory role in the further development of logical thought, since they contributed to the refinement of the basic forms of deductive reasoning and the systematisation of rules of inference. Despite the absence of a symbolic apparatus, the scholastic tradition laid the conceptual, broadly philosophical, and methodological foundations that were subsequently reconceptualised within early modern logic.

Further attempts at the modernisation of logic in the works of G.W. Leibniz

After the nominalist scholastics (or those close to them in their intentions and methodology), it is difficult to identify any clear changes in the position of logic until the beginning of the early modern period. The next stage in the development of logic may be found in G.W. Leibniz (1969), who in the seventeenth century not only added a fourth logical law (though it is not purely formal) but was in all likelihood the first in history to seek to reform logic in a thoroughgoing manner on the basis of ideas concerning its mathematisation. G.W. Leibniz held that logic requires the precision and computability of arithmetic and that its laws would only further the development of logic as a science; and it was precisely such propositions that became the foundation for attempts to introduce the fundamental principles of mathematics into logic. The works of G.W. Leibniz in the field of logic were ahead of their time, and in various contemporary forms of logic one frequently encounters elements that were created by him long before the definitive mathematisation of logic. For instance, G.W. Leibniz & F. Schmidt (1960) formulate the law of identity, which is employed to this day in many contemporary forms of logic and which became the principle of the intersubstitutability of formulas whose values correspond to one another (also known as the "principle of substitution of equivalents"). It is expressed as follows: "If A is B and B is A, then A and B are called 'identical'". Or: "A and B are identical if they can be substituted for one another". Speaking directly of G.W. Leibniz's logic, it would be difficult to identify any single coherent symbolic system, since he constantly experimented and employed a great many different approaches. A part of G.W. Leibniz's logical apparatus was in many respects a proto-variant of the modal logics known today and contained four principal categories: truth, falsity, possibility, and necessity. Truth and falsity, as in subsequent modal logics, were represented by the symbols T and F, and possibility and necessity by the symbols ◊ and □. For instance, the proposition "The sun rises" may be expressed by means of the symbol "S", signifying "The sun rises". Then "The sun rises" may be expressed as "S□T", meaning "The sun rises and this is a necessary truth". G.W. Leibniz (1969) not only anticipates the foundations of the future modal logics of deontic and alethic modalities but also, proceeding from classical syllogistic, continues to develop his symbolic system in a comprehensive manner, adding new signs and new types of connectives; his logic is, in general, replete with a great many different approaches and experiments.

In a discussion of the grounds for such transformations in logic, it is important to note that for G.W. Leibniz knowledge as such reduces to the proof of propositions, and that these proofs must be discovered by means of a definite method; it was precisely in logic that G.W. Leibniz saw an instrument for analysing any proposition and for conducting correct reasoning in any domain of activity whatsoever. In G.W. Leibniz's (1969) interpretation, logic appears not only as an auxiliary instrument of cognition but also as the methodological foundation of philosophical thought. The thinker associated its significance with the fact that it formulates the general rules for distinguishing the true from the false and creates the basis for the consistent demonstration of conclusions on the basis of definitions and experience. With such an understanding of logic and its purpose, logic itself would inevitably appear first and foremost as the universal and indispensable language of all science; but for this purpose it stood in need of mathematical rigour and computability, and it was precisely this logic that G.W. Leibniz sought to create, endeavouring to extend the symbolic component of formal logic. Thus, G.W. Leibniz was the first to advance the idea of reducing logic to mathematics, thereby continuing W. Ockham's work of refining and clarifying classical syllogistic. In doing so, he broadly anticipated the approaches of future logics; however, unlike the outstanding logicians of the twentieth century – such as G. Boole, G. Frege, or B. Russell – G.W. Leibniz did not set himself the task of demonstrating the narrowness of Aristotle's traditional logic. For G.W. Leibniz, syllogistic and mathematical logic served entirely different purposes and in no way impeded one another. It was for this reason that G.W. Leibniz sought to clarify and simplify Aristotle's logic whilst simultaneously not abandoning attempts to create an entirely new system of propositional calculus employing the laws and principles of mathematics. These propositions already give clear expression to the approach that would later be developed first by logicism and subsequently inherited by logical positivism: logic is understood as a form of the general rational organisation of knowledge, rather than as an auxiliary part of rhetoric or scholastic propaedeutics. It is for this reason that G.W. Leibniz may be regarded as one of the most important precursors of both logicism and the very idea of a unified language of science. However, despite the great significance of G.W. Leibniz's works on logic, almost all of them were, as is well known, lost until the mid-twentieth century and lay for centuries in piles of manuscripts in the royal library in Hanover; consequently, in the course of the future renewal of logic, all of his ideas and beginnings had to be, as it were, rediscovered by researchers such as G. Frege or C.S. Peirce.

In general, in order to broaden the context, it should be noted that one of the principal philosophical prerequisites for such an understanding of logic was also the long-running debate between rationalist and empiricist conceptions of cognition. The reference is to the protracted dispute between rationalist and empiricist conceptions of cognition. Classical empiricism (F. Bacon, J. Locke, D. Hume) proceeded from the thesis that the source of knowledge is sensory experience and that thought is of a secondary, derivative character: it merely generalises data obtained through observation and induction. From this perspective, logic was understood rather as an instrument for ordering already available experience than as an independent source of new truths. The rationalist tradition (R. Descartes, B. Spinoza, G.W. Leibniz), by contrast, proceeded from the opposite intention: there exists a level of knowledge that cannot always be reduced to empirical data and that may be obtained through rational analysis and deductive derivation. In this context, the position of G.W. Leibniz is of particular importance: he regarded the intellect as a system of operations capable of functioning according to formal rules, independently of the content of particular experience. The key idea of G.W. Leibniz in this regard was the concept of the characteristica universalis – a universal symbolic language that would make it possible to reduce any act of reasoning to a computational procedure. In conjunction with the project of the calculus ratiocinator, this signified an aspiration to transform logical reasoning into a kind of "calculus of truth", in which disputes between philosophers could be resolved not by argumentation in natural language but by formal computation. In this way, thought begins to be conceived not merely as a psychological or contentual process but as an operation on symbols according to given rules. It was precisely this orientation that laid the foundation for the subsequent attempts at the formalisation of logic in the nineteenth and twentieth centuries, when G. Frege, C.S. Peirce, E. Schröder, and other thinkers developed the idea of logical calculus, thereby in effect realising a part of the Leibnizian programme. In sum, it was precisely the tension between the empiricist conception of knowledge as the inductive generalisation of experience and the rationalist idea of the autonomy of formal thought that created the intellectual space within which the emergence of mathematical logic as a distinct discipline became possible.

If one returns to the logical investigations of G.W. Leibniz & F. Schmidt (1960) and G.W. Leibniz (1969) in a broader context, it must be noted that, despite the considerable number of innovative ideas, it is difficult to find a fully completed and systematically expounded logical system in his works. G.W. Leibniz's logical reflections are presented in a large number of fragments, projects, and drafts devoted to the creation of a universal symbolic language and a logical calculus. Even if one regards these contributions as elements of a single "Leibnizian logic", its expressive possibilities remained limited in comparison with the subsequent forms of mathematical logic. G.W. Leibniz himself was aware of this circumstance and repeatedly emphasised the necessity of further perfecting the logical apparatus. One of the principal differences between G.W. Leibniz's logical projects and contemporary predicate logics is their limitation in representing complex relations between objects. Logical analysis in G.W. Leibniz remained, to a considerable degree, bound to the traditional subject–predicate structure of the judgement inherited from the Aristotelian tradition. As a consequence, his logical schemata were less suited to the analysis of polyadic relations, quantified propositions, and complex logical constructions. Only in the subsequent works of C.S. Peirce, E. Schröder, G. Frege, and his followers were formal systems created that made it possible to carry out logical analysis not only of individual objects but also of properties, relations, and logical structures of considerably greater complexity.

Analysis of the process and grounds for the creation of contemporary mathematical logic

After G.W. Leibniz, the process of the renewal of logic begins in the mid-nineteenth century, specifically with the works of G. Boole, C.S. Peirce, A. de Morgan, G. Peano, and G. Cantor, amongst others, in which the new mathematical logic finds its initial foundations. However, these approaches were not homogeneous either in their subject matter or in their aims: whilst Boolean algebra and the formalisation projects of G. Peano and G. Cantor were developed primarily within the framework of foundational mathematics and its axiomatisation, another line of development was directed at the analysis of the very structure of the logical sign, its functioning, and modes of interpretation (Scholz, 1931). An important philosophical background to this divergence of directions was the conflict between psychologism and anti-psychologism in the understanding of the nature of logical laws. The psychologistic position, which was widespread within the empiricist tradition of the nineteenth century, treated logic as a generalisation of the actual processes of thought, thereby reducing logical forms to the sphere of mental acts. From this perspective, the laws of logic lost their objective and normative character, since they were regarded as descriptive generalisations of how a subject actually thinks. The anti-psychological line, by contrast, which gradually gathered force in the second half of the nineteenth century – above all in the works of G. Frege – proceeded from a principled distinction between acts of thought and the content of thought as an objective structure (Scholz, 1931). In this interpretation, logical laws do not depend on the empirical particularities of the psyche but possess the status of universal norms of correct reasoning, operative independently of individual or collective experience. It was precisely this opposition that became the conceptual prerequisite for the transition from psychologically oriented interpretations of logic to its formalisation as an autonomous deductive calculus.

The elimination of psychological content from logical analysis made it possible to focus on the purely formal relations between signs and on the rules of their transformation, which became the decisive step in the formation of contemporary mathematical logic. In this context, the algebraic tradition of G. Boole and A. de Morgan, as well as the relational logic of C.S. Peirce (1873), may be regarded as intermediate stages that had not yet fully broken with psychological interpretations but had already substantially prepared the ground for the anti-psychological turn. The definitive conceptual consolidation of this turn occurs in the Fregean programme of logicism, in which logic appears as a system of objective meanings and formal rules of inference, independent of any mental processes. In this context, the contribution of C.S. Peirce acquires particular significance: building upon the development of the algebraic logic of G. Boole and A. de Morgan, he substantially broadened its problematic field and – bringing logic closer to a modern formal analysis of thought and sign systems (in parallel with G. Frege in the domain of logical investigations) – proposed a fundamentally broader approach to logical forms, examining them in connection with a general theory of signs (semiotics) and a pragmatic interpretation of meaning. It is in C.S. Peirce (1873) that logic begins to be understood not merely as a computational system but as a universal theory of relations between signs, objects, and interpreters, which substantially broadens the perspective of the mathematisation of logical thought. The contribution of C.S. Peirce to the renewal of logic is determined above all by the fact that he was among the first to consistently take logical analysis beyond the limits of the traditional syllogistic and to give it the form of a formal calculus. For C.S. Peirce (1960), the task consisted not in the partial improvement of the Aristotelian schema but in a substantial extension of logic beyond the limits of the traditional terminological syllogistic. C.S. Peirce, building on Boolean calculus and the contributions of A. de Morgan, constructs a logic of relations as a broader formal apparatus for expressing dependencies and modes of inference. In this way, logic begins to be understood not as a collection of ready-made figures of syllogism but as a general symbolic system for the analysis of formal connections. It was precisely this transition from terminological logic to a logic of relations that became one of the decisive conditions for the modernisation of logic in the second half of the nineteenth century. It may be noted that the investigations of C.S. Peirce were in very close connection with the works of E. Schröder, and this connection is quite clearly documented: C.S. Peirce's later works after 1890 contain numerous references to E. Schröder and are frequently accompanied by an extended discussion of his logical ideas; moreover, their direct correspondence is well attested (Houser, 1990). At the same time, G. Frege's investigations (1903), which chronologically developed in parallel with the works of C.S. Peirce (1873), may be regarded with a certain caution as independent of them; similarly, the logical contributions of C.S. Peirce should not be directly derived from the Fregean tradition.

If one poses directly the question of whether there exists a direct connection between C.S. Peirce and G. Frege, the answer is most likely negative: at least in the known publications and in the customarily analysed corpus of G. Frege's texts (1903; 1967), no such references have been recorded; however, it is important to emphasise that this qualification is not equivalent to a strict proof that G. Frege could not have known of C.S. Peirce and his logical contributions at all. In the article of B.S. Hawkins (1993), the problem is directly characterised as "unanswered": the absence of direct references is noted, but at the same time indirect circumstances are examined that do not allow the question to be definitively closed. Nevertheless, what is characteristic of the investigations of both C.S. Peirce and G. Frege is that, in a broad philosophical sense, they jointly associated the transition from terminological logic to a logic of relations with its systematic mathematisation. C.S. Peirce (1960) directly formulates the task of constructing an algebra adequate for the resolution of all problems of deductive logic. Logic acquires not only a more rigorous symbolic form but, properly speaking, an operational character: C.S. Peirce develops ways of operating on logical propositions as formal objects admitting of transformation according to general rules. The most important step was the introduction of operations on relations – above all relational addition (union) and relational multiplication (composition of relations) – by virtue of which the subject of logical analysis becomes not only classes and predications but also relational structures. This signified a substantial increase in the expressive power of logic and simultaneously its approximation to the mathematical style of thought, in which the central role is played not by the linguistic description of forms of inference but by the calculus of admissible transformations.

It is no less important that it is in C.S. Peirce (1960) that the logic of relations gradually passes into a more developed quantificational form. He develops means that allow one to systematically express the distinction between universality and existence – that is, between what would later be standardly captured through the universal and existential quantifiers. It is noteworthy that in the text of 1885, C.S. Peirce (1960) already explicitly employs forms of the type "Any" and "Some", extending them to both individual predications and the domain of relations. It is for this reason that in the contemporary historiography of logic – in particular in the work of W. Ewald (2019) – C.S. Peirce's investigations of 1870-1885 are regarded as one of the important stages in the formation of first-order logic. It is important that quantification in C.S. Peirce appears not as an external supplement to the preceding logical tradition but as the natural result of an attempt to formalise relational propositions that could not be adequately described by the means of classical syllogistic. In general, the influence of C.S. Peirce on the renewal of logic may be reduced to several interrelated results. First, he radically expanded the subject matter of logic, placing relations – rather than merely terms and classes – at its centre. Second, he gave logic the form of a rigorous calculus oriented towards formal operations and transformations. Third, he developed means of expressing quantification, thereby bringing logic closer to its modern and familiar predicate form. Finally, he showed that logical rigour can be achieved not only in linear symbolic notation but also in diagrammatic systems. C.S. Peirce should therefore be regarded not as a secondary figure in the transition from traditional to modern logic but as one of the authors of that transition (alongside G. Frege), whose works constitute an independent and extraordinarily influential line of the mathematisation of logic. However, notwithstanding the substantial weight of the investigations of C.S. Peirce or E. Schröder in the formation of algebraic and symbolic logic, it is precisely from the works of G. Frege that a fundamentally new stage begins – the stage of a genuine mathematisation of logic in the strict, systematic sense. Whereas previous approaches for the most part developed logic as a variety of algebraic computation or as an improvement of formal procedures of reasoning, in G. Frege logic appears for the first time as the universal foundation of all mathematics.

An important philosophical prerequisite for the final line of the mathematisation of logic may be identified in the broad conflict between psychologism and anti-psychologism that unfolded at the end of the nineteenth century. Representatives of psychologism regarded logical laws as generalisations of the actual processes of human thought. In this context, C.S. Peirce and E. Schröder were not anti-psychologists in the strict Fregean sense, but neither did they reduce logic as a whole to psychology. The position of C.S. Peirce in particular is intermediate and more complex. He in fact criticised the reduction of logic to individual psychology but did not separate it entirely from the theory of thought. For C.S. Peirce (1960), logic is not a description of the mental states of the individual but a normative and formal theory of signs and inference (semiotics), whilst at the same time being connected to the general laws of thought as a process. That is, thought in his account is not "psychological" but semiotic-inferential (through signs and conclusions). G. Frege (1903), by contrast, consistently substantiated the independence of logical laws from the psychological mechanisms of consciousness and thought, regarding them as objective and universally valid (akin to mathematical ones). The aspiration to liberate logic from psychological elements contributed to the development of formalised logical systems and became one of the important prerequisites for its definitive mathematisation, above all in the works of G. Frege and subsequent logicists. G. Frege undertook a large-scale and independent reconstruction of logic in the works "Begriffsschrift" and "The Foundations of Arithmetic", as part of attempts to reduce formal logic to mathematics (Shramko, 2005a). In the works of Y. Shramko (2005a; 2005b), it is emphasised that G. Frege's contribution was of fundamental significance for the formation of modern logic. His innovation lay in the rejection of the limitations of Aristotelian syllogistic, the construction of a deductive system as a predicate calculus, and the introduction of a functional analysis of language in place of the traditional subject–predicate schema.

If the initial attempts to reconstruct formal logic find their prerequisites in the works of G.W. Leibniz, G. Frege rediscovers these beginnings afresh, seeking to reduce logic to mathematics and thereby initiating the process of its transformation. The philosophical foundations of the renewal of logic in G. Frege (1903) are connected with his aspiration to substantiate arithmetic as a part of logic, independent of empirical experience or intuitive intuition. On this basis, he argued for the possibility of deriving the fundamental laws of arithmetic by purely logical means. In this way, G. Frege's anti-psychological orientation receives concrete expression in his logicist programme. G. Frege formulates here an even stronger thesis than mere anti-psychologism: arithmetic is interpreted as a part of logic – that is, as a system of propositions that require no support from experience or intuition whatsoever. It is important that he not only separates logic from psychology but also from empirical cognition in general, including intuitive or spatiotemporal intuition. It is for this reason that the mathematisation of logic in G. Frege appears not as a technical formalisation but as a consequence of a radical reconceptualisation of the nature of logical and mathematical truth. It is on the basis of precisely these premises that G. Frege substantiates logicism and creates a new axiomatics, enumerating the logical rules by means of which new concepts are defined and theorems proved. Thus, in the works of G. Frege, a fundamentally new stage in the development of logic is realised, connected with its radical formalisation and transformation into an autonomous deductive calculus. As E.D. Walker & E.H. Reck (2024) noted, it was in the "Begriffsschrift" that a systematic symbolic language was first proposed that made it possible to capture the logical structure of judgements beyond the boundaries of the grammar of natural language or epistemological appeals to the psyche and to derive propositions by means of strictly defined rules of deduction. In this sense, G. Frege did not so much "axiomatise propositional logic" in the modern understanding as create the first fully developed system of predicate logic with quantification, in which the propositional level is already implicitly included as a particular case of a more general structure. At the same time, the formation of propositional calculus as a relatively autonomous formal fragment of logic appears as the result of a subsequent stage of development, connected above all with G. Boole's Boolean algebra, A. de Morgan's logic of relations, and the contributions of C.S. Peirce and E. Schröder, in which operations on propositions acquire an algebraic form. In these investigations, the propositional level of formalisation proper is formed, though still without the complete axiomatic autonomy characteristic of later systems.

The further crystallisation of propositional logic as a completed axiomatic system occurs already in the context of the logicism of A.N. Whitehead & B. Russell (1925), in which the "Principia Mathematica" effects a systematic inclusion of propositional calculus as the base layer of formal deduction. In the later terminology, this level is interpreted as zeroth-order logic, which serves as the fundamental basis for the construction of higher-order predicate calculi (Mancosu, 2008). Formally, propositional logic is specified as an inductively defined set of formulae constructed from propositional variables *p*, *q*, *r* by means of a finite set of logical operations. Atomic formulae are identified with propositional variables, whilst compound formulae are formed recursively through the application of the operations of negation, conjunction, disjunction, implication, and equivalence. Semantically, these formulae are interpreted as propositions that take one of two truth values in accordance with the principle of bivalence. The logical operations are thereby specified as functions over the set of truth values {0,1}, which provides an algebraic interpretation of the propositional calculus. The syntactic structure of the formal language is determined through the rules for constructing well-formed formulae, and the use of parentheses serves as a means of fixing the hierarchy of operations and eliminating syntactic ambiguity. As a result, propositional logic appears as a closed formal system in which the syntactic rules of construction and the semantic interpretation stand in a strictly determined correspondence, which ensures its role as the base level of contemporary mathematical logic. Table 1 summarises the key aspects of the difference between propositional logic and predicate logic.

***Table 1.** Principal differences between propositional logic and predicate logic as two levels of formal logic*

Parameter	Propositional logic	Predicate logic (order logics)
Principal object	Entire propositions	Objects, properties, and relations
Structure of the proposition	Unanalysable (atomic)	Internal structure (predicate + arguments)
Basic elements	*p*, *q*, *r* (propositional variables)	Predicates $P(x)$, $R(x, y)$, functions

Table 1. *Continued*

Parameter	Propositional logic	Predicate logic (order logics)
Variables	Absent	Individual variables (x, y, z)
Quantifiers	None	∀ (universal), ∃ (existential)
What it describes	Truth of entire propositions	Structure of objects and their properties
Semantics	Truth tables	Interpretations over a domain of objects
Expressive power	Limited	Substantially greater
Example	$p \wedge q$	$\forall x\ (P(x) \rightarrow Q(x))$
Role in logic	Base level of formalisation	Foundation of contemporary mathematical logic

Source: *compiled by the author on the basis of H.B. Enderton (2001)*

Thus, propositional logic came to be understood as one of the two principal parts of contemporary mathematical logic, the second part being the predicate order logics, which differ from propositional logic in that propositional logic focuses primarily on the analysis and formalisation of the truth or falsity of individual propositions; this logic works exclusively with atomic propositions that are not decomposed into smaller constituents; and propositional logic does not account for the entire internal structure of propositions and their possible semantic interrelations. Order logics are more complex and diverse systems that make it possible to express relations between different objects and to account for the order or structure of these objects. In predicate logic, as is well known, there are also quantifiers, which serve the function of restricting the domain of truth of predicates and can account for relations – "includes/introduces", "precedes/follows", and so on; and the quantifiers themselves are equally rightfully regarded as a direct discovery of G. Frege. One may say that predicate order logics are more flexible and in essence present themselves as extensions of propositional logic, since their axiomatics and semantics are very similar. Propositional logic is oriented primarily towards the analysis of the truth of propositions and their calculus, whilst order logics are more flexible and are designed to analyse more complex relations and structures. It is precisely these two key divisions that to this day are understood as the two principal modes of constructing mathematical logic and constitute what is customarily referred to as mathematical logic (Hurley & Watson, 2018)

It is important that such undertakings on the part of the leading mathematicians of the era, the foundations of such approaches to logic, and above all the achievements of G. Frege himself, long remained not fully understood by the scholarly community, and the very idea of reducing logic to mathematics and the entire process of constructing propositional and predicate calculi were received less enthusiastically than might initially appear. Only a small number of researchers – such as A. Meinong or B. Russell – appreciated the full importance and relevance of G. Frege's work and shared his aspirations in the context of logic and mathematics. In European academic philosophy, the process of the modernisation of logic gained wide currency only in the first decades of the twentieth century, above all after the publication of the "Principia Mathematica", in which A.N. Whitehead & B. Russell (1925) continue and develop G. Frege's work, offering a more perfected variant of the systems of propositional logic and predicate order logics. The innovative character of G. Frege's logical programme is also emphasised in the contemporary historiography of logic. In particular, W. Ewald (2019) draws attention to the fact that the significance of Frege's contribution did not become evident immediately but only in the subsequent development of formal logic.

The full significance of the ideas laid down in G. Frege's works was revealed not at once but only in the subsequent perspective of the development of mathematical logic. It was this development that gradually disclosed the historical significance of his contribution to the formation of contemporary formal logic. In 1931, for instance, the eminent German logician and historian of logic H. Scholz (1931) wrote that G. Frege is unquestionably the most outstanding of the logicians of the nineteenth century. Until the first decades of the twentieth century, the renewal of logic was carried out for the most part by the efforts of individual researchers and had not yet acquired the character of a broad intellectual movement. A decisive role in the transformation of this status was played by the dissemination of the logicist programme, the formation of the Vienna Circle, and the reception of the "Principia Mathematica" of A.N. Whitehead & B. Russell (1925). Continuing the ideas of G. Frege, this work demonstrated the new possibilities of formal logic and became an important source for logical positivism and cognate currents of logical empiricism, in particular H. Reichenbach's Berlin Circle. Logic occupies a truly prominent place in the philosophical programme of the Vienna Circle and in the currents of logical positivism and logical empiricism in general (Whitehead & Russell, 1925). In effect, it was in the new logic that the neo-positivists found the instrument that was to become the principal means of analysing science, and the creation and development of mathematical logic laid the foundations of a new ontological picture of the world and of all subsequent analytic philosophy.

At the beginning of the twentieth century, mathematical logic in all the diversity of its forms replaced classical syllogistic and came to be widely employed in many areas and forms of scientific activity. Broadly speaking, mathematical logic presents itself as a more general formal method that employs symbols and algebraic operations for the construction and analysis of logical expressions. It encompasses formal symbolism and structures that allow for a more precise investigation of various types of logical relations. The foundations and grounds of the reconstruction of logic consist in attempts to construct a formal system with the precision, clarity, and computability characteristic of mathematics – reducing logic and the fundamental propositions of mathematics to a single framework – and these foundations and propositions echo analogous positions of earlier logicians such as the aforementioned W. Ockham (1957) or G.W. Leibniz (1969). V. Kraft (1953) connected the substantial renewal of logic with the activity of mathematicians, for whom traditional logic no longer provided sufficient means for the justification and construction of mathematics. The propositions of mathematics do not fit the schema of the judgements of traditional logic – subject-copula-predicate – because they express connections and relations. "Propositions that relate a single subject to a single predicate are suitable only for properties, for classes, but they cannot be used to express relations that connect two or more elements." The grounds for the renewal of logic, the genealogy of which may be traced back to the era of scholasticism, may therefore be identified as the extreme and excessive unwieldiness of traditional logic, which, in combination with its limitations and its inability to work with many elements, leads to the necessity of developing a new logical theory of relations adequate to the requirements of contemporary science and reflecting the latest achievements in mathematics. These ideas are manifested in the general aspiration to demonstrate the possibility of reducing logic to mathematics, which consists in the construction of a mathematical formal system capable of serving as the foundation for logic. Such an approach makes it possible to employ the methods of mathematics for the analysis and investigation of logical systems and substantially broadens the understanding not only of the possibilities of logic but also of the fundamental foundations of mathematical knowledge, in effect demonstrating a new stage of the philosophy of mathematics.

As V. Kraft (1953) noted, the need for the reform of logic was conditioned not only by the development of mathematics but also by the emergence of logical difficulties and antinomies that revealed the general limitations of traditional logical means. The existence of a multitude of antinomies and paradoxes that impeded the further development of logic also became one of the key reasons for the creation of Russell and Whitehead's theory of types and for the extensive work carried out within the framework of logic and mathematics. In the "Principia Mathematica", A.N. Whitehead & B. Russell (1925) defined mathematical logic as a systematically constructed foundation of inquiry, the first part of which was to realise several key theoretical aims. First, it aimed to effect the most comprehensive possible analysis of the concepts it employs and of the processes by which it conducts proofs, and to reduce these to the smallest possible number of undefined concepts and unproved propositions (called, respectively, primitive ideas and primitive propositions) from which it proceeds. Second, it was created for the most precise possible expression of mathematical propositions. Third, A.N. Whitehead & B. Russell (1925) emphasised that their logical system was specifically directed at overcoming the paradoxes that in preceding years had become one of the central problems of symbolic logic and set theory. They connected the possibility of avoiding such contradictions with the theory of types, which was to eliminate the imprecisions that led to the emergence of logical antinomies.

In speaking of paradoxes, A.N. Whitehead & B. Russell had in mind self-descriptive semantic paradoxes – a class of general logical paradoxes arising in connection with self-description (the description of themselves) within both the formal language of logic and mathematics and within ordinary language. One of the most well-known self-descriptive semantic paradoxes is Russell's paradox, which had long before been known as the "liar's paradox" and, in its current formal guise, arose precisely in the context of set theory and self-descriptive propositions, and which impeded the consistent and non-contradictory construction of mathematics within the new logic. After discovering the relevant contradiction, B. Russell informed G. Frege that it reveals a problem in the logicist system underlying the Fregean project of justifying arithmetic. G. Frege's attempts to eliminate this inconsistency did not yield a convincing result, and the subsequent development of the problem was substantially connected with the "Principia Mathematica" of A.N. Whitehead & B. Russell (1925), which proposed a new formal apparatus for avoiding such logical antinomies. The "Principia Mathematica" became the most fundamental and comprehensive work on logic and mathematics in history. Following its republication in 1925, it gained wide popularity and recognition amongst logical positivists, and it was broadly through this work that the new mathematical logic gained currency. It was precisely A.N. Whitehead & B. Russell (1925) who demonstrated all the possibilities of predicate logic whilst simultaneously persuading the scholarly community of the utility and universality of the idea of formal systems. In its new form, logic makes it possible to perform exclusively formal operations on propositions and concepts, and by means of the new logic a clarity and precision is achieved such as could not have been contemplated when employing ordinary (everyday) language.

In the contemporary historiography of mathematical logic, there exists a number of interpretations that explain the character and mechanisms of its formation in different ways. Within the framework of the present study, the mathematisation of logic was regarded as a multi-factorial process in which the algebraisation of logical operations, the development of the theory of relations, the formalisation of language, and logicism as an attempt at the reduction of mathematics to logic were all combined. The conclusions obtained allow this position to be compared with the approaches of other researchers. In the work of J. van Heijenoort (1967), the development of logic is interpreted as a sequential evolution of formal systems in which the decisive moment is the transition from the algebraic tradition of the nineteenth century to rigorously axiomatised systems. This approach is broadly consistent with the results of the present study regarding the key role of formalisation; however, it somewhat underestimates the significance of C.S. Peirce's relational logic and its influence on the subsequent development of predicate structures. M. Dummett (1981), in analysing G. Frege's legacy, emphasises the semantic turn as the central moment in the formation of modern logic. In contrast to this, the present work shifts the emphasis to the institutional-methodological aspect of mathematisation, in which what is important is not only the semantic innovations but also the structural integration of logic with mathematics through symbolic systems and the shared philosophical arguments of this process. G. Gabriel & W. Kienzler (1997) regarded G. Frege's logicism as a radical break with the preceding tradition of psychologism and empiricism. In the present study, this position is partially corrected: G. Frege's anti-psychologism is acknowledged as a key prerequisite, but its continuity with respect to the algebraic logic of G. Boole and A. de Morgan is emphasised, which attests rather to a transformation than to a complete break. This approach partially coincides with the conclusions of the study, but it insufficiently accounts for the role of the contributions of C.S. Peirce and E. Schröder as an intermediate and authentic link between algebraic and Fregean logic. P. Mancosu (2008), in analysing logicism and its limits, emphasises the internal tension between the programme of reducing mathematics to logic and the development of G. Cantor's set theory. In the present study, this tension is also acknowledged, but it is additionally emphasised that it was precisely the synthesis of logicism and the set-theoretic mathematical approach of A.N. Whitehead & B. Russell (1925) that proved decisive for the final formation of contemporary mathematical logic. In general, the results of the study are consistent with the principal tendencies of contemporary historiography regarding the recognition of the multi-component character of the formation of mathematical logic, but they differ in a more pronounced emphasis on the genealogical aspect and on the continuity of the transition between algebraic, relational, and logicist programmes upon a generally shared philosophical foundation.

Conclusions

As a result of the historico-philosophical analysis conducted, it was established that the process of the renewal and mathematisation of logic constituted a prolonged, multi-stage development whose roots reach back to the late scholastic period. Already in the works of W. Ockham and his followers, a critical attitude towards the excessive complexity of the traditional Aristotelian syllogistic takes shape and the need for the refinement, simplification, and formalisation of rules of inference is recognised. These intentions laid the proto-formal prerequisites for the subsequent transformation of logic and the foundation for the future philosophical grounds of the mathematisation of logic. An important stage on the path towards contemporary mathematical logic was constituted by the projects of G.W. Leibniz, who was the first to advance a programme for the mathematisation of logic and the creation of the characteristica universalis – a universal symbolic language of science capable of reducing reasoning to a computational procedure. Although these ideas remained fragmentary, they influenced the formation of a rationalist paradigm within which logic began to be regarded as an autonomous formal instrument of cognition, independent of empirical experience. In the nineteenth century, the mathematisation of logic accelerates within the algebraic tradition initiated by G. Boole and A. de Morgan. They introduced an algebraic approach to logical operations that made the formal calculus of propositions possible. This line of development was extended by C.S. Peirce, who moved from the logic of classes to the logic of relations, developed relational operations and means of expressing quantification. His works, together with those of E. Schröder, formed an influential line in the mathematisation of logic that substantially expanded the expressive possibilities of formal systems and prepared the ground for contemporary predicate logic. The decisive transformation in the development of logic was effected by G. Frege, who created the first fully developed system of predicate logic with quantification, carried out the axiomatisation of propositional logic, and substantiated logicism – the reduction of arithmetic (and subsequently of all mathematics) to logic. His philosophical arguments in favour of the reduction of logic to mathematics, his anti-psychologism, his functional analysis of language, and his rejection of the subject-predicate structure became the fundamental grounds of contemporary formal logic. The further development of these ideas in the "Principia Mathematica" consolidated mathematical logic as an independent discipline and made it possible to resolve a number of paradoxes (in particular Russell's paradox) that had arisen in set theory. By means of the theory of types and systematic axiomatisation, logic acquired the necessary rigour, precision, and expressive power that surpasses the possibilities of classical syllogistic. Thus, the mathematisation of logic was conditioned by a complex of philosophical, mathematical, and methodological factors: the limitations of syllogistic with respect to complex relations, the need to overcome paradoxes, the conflict between psychologism and anti-psychologism, and the aspiration towards a universal rational language of science. In sum, mathematical logic (propositional logic and predicate logics) became the foundation of contemporary mathematics, computer science, and artificial intelligence, as well as a key instrument of the philosophical analysis of scientific rationality. Further research may be directed at the analysis of the influence of mathematical logic on contemporary analytic philosophy, the philosophy of science, and current formal methods in the humanities.

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Підстави оновлення і математизації логіки та їх історична характеристика

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Анотація. Актуальність дослідження зумовлена необхідністю історико-філософського осмислення процесу оновлення та математизації логіки, що визначив формування сучасної математичної логіки та суттєво вплинув на розвиток філософії науки, математики й аналітичної традиції. Метою статті було висвітлення філософських підстав, аргументів і концептуальних передумов, які сприяли реформуванню класичної логіки та становленню її математичних форм у другій половині XIX – на початку XX століття. Методологічну основу дослідження становили історичний, аналітичний, порівняльний та генеалогічний методи, що дозволяють простежити еволюцію уявлень про логіку від схоластичної традиції до сучасних форм математичної логіки. Також використано метод формалізації для аналізу особливостей логіки висловлювань і предикатних логік у контексті їх відмінностей. У результаті дослідження встановлено, що перші підстави модернізації логіки формуються вже в межах схоластичної традиції, зокрема у працях В. Оккама, де обґрунтована необхідність уточнення та спрощення силогістичних правил. Показано, що важливим етапом розвитку цих ідей стали логічні проєкти Г. В. Лейбніца, який одним із перших висунув програму математизації логіки та створення універсальної символічної мови науки. Визначено, що вирішальну роль у реформуванні логіки відіграли дослідження Г. Фреге, який заклав основи логіцизму, здійснив аксіоматизацію логіки висловлювань і створив засади сучасного числення предикатів. З'ясовано, що подальший розвиток цих ідей у працях Б. Рассела та А. Уайтхеда сприяв утвердженню математичної логіки як самостійної галузі знання та універсального інструменту наукового аналізу. Також встановлено, що важливими чинниками математизації логіки стали обмеженість класичної силогістики та необхідність подолання логічних парадоксів, які виникли в процесі розвитку математики. Практична цінність дослідження полягає у можливості використання його результатів для історико-філософського аналізу логіки, викладання курсів з логіки, філософії науки та історії аналітичної філософії, а також для подальших досліджень генези математичної логіки

Ключові слова: позитивізм; логіцизм; пізнання; силогістика; зчислення предикатів; квантор; судження